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High-Frequency Sum-Rule Expansion for Relativistic Quasi-One-Dimensional Quantum Plasma Dielectric Tensor

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A high-frequency sum rule for all elements of the relativistic spinless quasi-one-dimensional quantum plasma response tensor at $T=0$ K is derived. It is found that the frequency of oscillations is reduced by the relativistic effect.

1. INTRODUCTION

High-frequency sum-rule expansions of the full response tensor of nonrelativistic and relativistic quantum plasmas in the absence of magnetic field are known (Genga, 1988a, 1992a,b). However, in the presence of an external magnetic field the only known results are those applicable to nonrelativistic situations (Genga, 1988b, 1989).

In this work I consider the high-frequency behavior of the full dielectric tensor in an anisotropic system of a relativistic quantum plasma with spinless particles of density 10^{29} particles per unit volume at $T=0$ K in the presence of an external magnetic field up to order ω^{-5}. The work is related to the Malmberg–O'Neil experiment where a strongly coupled electron plasma is generated.

In laboratory plasmas, unlike in the astrophysical case, the radiation effect is not appreciable, and hence is negligible. I apply the Hamiltonian formalism to derive the high-frequency sum rules as in the nonrelativistic case. Further, as in the magnetic-free case (Jancovici, 1962; Genga, 1992b) an electron may jump from one state inside the Fermi sphere to an unoccupied state due to interaction, thus leading to the creation of a "Fermi hole" behind. The jump of an electron from a negative-energy state to an

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occupied positive-energy state leads to the creation of a positron–electron pair; the positron is a hole that is called a "Dirac hole." In this case the interaction is a purely Coulomb one (Goldstone, 1957). The system is therefore described by a set of unperturbed states which allow for positrons and electrons.

The method of derivation is reviewed below. In Section 2 the general relationships between the external or current–current response function sum-rule coefficients and those of the dielectric tensor are reviewed and the exact ω^{-2}, ω^{-3}, ω^{-4}, and ω^{-5} sum-rule coefficients for the transverse element are obtained. The long-wavelength limit of the results and their possible implications for the dispersion relation of plasma modes are considered in Sections 3 and 4, respectively.

The total electron current at point X_1 is given by

$$j(x_i)=\frac{e}{2}\sum\left[\mathbf{V}_i\delta(\mathbf{x}-\mathbf{x}_i)+\delta(\mathbf{x}-\mathbf{x}_i)\,\mathbf{V}_i\right) \tag{1}$$

where $\mathbf{V}_i$ is the group velocity of the particle i. The total energy of a free spinless particle is given by (Johnson and Lippman, 1949; Genga, 1992a,b; Berestetskii *et al.*, 1978; Baym, 1974; Sakura, 1987; Bjorken and Drell, 1964)

$$\mathbf{E}=(\Pi^2c^2+m^2c^4)^{1/2} \tag{2}$$

where

$$\begin{aligned}\Pi&=\mathbf{P}-\frac{e}{c}\,\mathbf{A}^0(\mathbf{r})-\frac{e}{c}\,\mathbf{A}(\mathbf{r}) \qquad \text{(the generalized momentum)}\\ \mathbf{A}^0&=\frac{1}{2}\,\mathbf{B}^0\times\mathbf{r} \qquad \text{(the external vector potential)}\end{aligned} \tag{3}$$

and $\mathbf{A}(\mathbf{r})$ is the self-consistent vector potential.

From equation (2) we find that

$$\begin{aligned}\mathbf{V}_i&=\frac{\partial\mathbf{E}(P)}{\partial\mathbf{P}}\\ &=\frac{\Pi c^2}{(\Pi^2c^2+m^2c^4)^{1/2}}\\ &=\frac{\Pi/m}{(1+\Pi^2/m^2c^2)^{1/2}}\end{aligned} \tag{4}$$

In Fourier transform language, this equation becomes

$$\langle J_{\mathbf{k}}^{\mu}(\omega)\rangle = e\langle j_{\mathbf{k}}^{\mu}(\omega)\rangle - \frac{e^2 N}{mc}\gamma^{-1}T_{\mathbf{k}}^{\mu\nu}A_{\mathbf{k}}^{\nu}(\omega) \tag{5}$$

where γ^{-1} is the relativistic term defined as

$$\gamma^{-1} = \frac{1}{(1+\Pi^2/m^2c^2)^{1/2}} = \left(1+\frac{u^2}{c^2}\right)^{-1/2} \tag{6}$$

with

$$\mathbf{u} = \Pi/m \tag{7}$$

as the nonrelativistic phase velocity of the particle. Equation (7) is arrived at after taking first the Fourier transform of equation (1) followed by its expectation value since we are interested in the response function of the electron system. By applying perturbation theory (Pines and Nozières, 1966; Genga, 1988a,b, 1989), it is found that

$$\begin{aligned}\langle j_k^{\mu}(\omega)\rangle = -\frac{e}{c}\sum_{np}\omega^{-1}\,\langle 0|\,\Pi_{\mathbf{k}}^{\mu}(\tau)\,|n\rangle\langle n|\,\Pi_{-\mathbf{k}}^{\nu}(0)\,|0\rangle \\ \times\left[\frac{1}{\omega-\omega_{n0}(p,\,p+\hbar\mathbf{k}/2)+i\eta}\right. \\ \left. -\frac{1}{\omega-\omega_{n0}(p,\,p-\hbar\mathbf{k}/2)+i\eta}\right]\mathbf{A}_{\mathbf{k}}^{\nu}(\omega)\end{aligned} \tag{8}$$

where

$$\Pi_{\mathbf{k}}^{\mu} = \tfrac{1}{2}\sum_i\,(\mathbf{V}_i^{\mu}e^{-\mathbf{k}\cdot\mathbf{x}_i} + e^{-i\mathbf{k}\cdot\mathbf{x}_i}\mathbf{V}_1^{\mu}) \tag{9}$$

with

$$\mathbf{V}_i^{\mu} = \gamma^{-1}\Pi_i^{\mu}/m \tag{10}$$

From equations (5) and (8) it is found that conductivity tensor $\sigma^{\mu\nu}$ becomes

$$\sigma^{\mu\nu}(\mathbf{k}\omega) = i\frac{e^2}{\omega}\left[\chi^{\mu\nu}(\mathbf{k}\omega) - \frac{N\gamma^{-1}}{m}\mathbf{T}_{\mathbf{k}}^{\mu\nu}\right] \tag{11}$$

where $\chi^{\mu\nu}(\mathbf{k}\omega)$ is the electron response tensor defined as

$$\chi^{\mu\nu}(\mathbf{k}\omega)=\sum_{np}\langle 0|\,\Pi^{\mu}_{\mathbf{k}}(\tau)\,|n\rangle\langle n|\,\Pi^{\nu}_{-\mathbf{k}}(0)\,|0\rangle \times\left[\frac{1}{\omega-\omega_{n0}(p,\,p+\hbar\mathbf{k}/2)+i\eta}-\frac{1}{\omega-\omega_{n0}(p,\,p-\hbar\mathbf{k}/2)+i\eta}\right] \tag{12}$$

In terms of the polarizability tensor, equation (11) becomes

$$\begin{aligned}\alpha^{\mu\nu}(\mathbf{k}\omega)&=i\,\frac{4\pi e^2}{\omega}\,\alpha^{\mu\nu}(\mathbf{k}\omega)\\&=\frac{\omega_p^2}{\omega^2}\,\gamma^{-1}\mathsf{T}^{\mu\nu}_{\mathbf{k}}+\bar{\alpha}^{\mu\nu}(\mathbf{k}\omega)\end{aligned} \tag{13}$$

where

$$\bar{\alpha}^{\mu\nu}(\mathbf{k}\omega)=4\pi e^2\,\frac{\chi^{\mu\nu}_{(k\omega)}}{\omega^2} \tag{14}$$

The matrix elements and excitation frequencies that appear in equation (14) are those appropriate for a system of electrons with Coulomb interactions.

2. TRANSVERSE SUM RULES

As in the nonrelativistic case, the complete modified polarizability tensor $\bar{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ is expressible in terms of corresponding "external" qualities $\hat{\bar{\alpha}}^{\mu\nu}(\mathbf{k}\omega)$,

$$\bar{\alpha}(\mathbf{k}\omega)=\hat{\bar{\alpha}}(\mathbf{k}\omega)(\Delta-\hat{\bar{\alpha}}(\mathbf{k}\omega))^{-1}\,\Delta \tag{15}$$

where

$$\begin{aligned}\Delta&=\mathbb{1}-n^2\mathsf{T}\\n&=\frac{kc}{\omega}\\\mathsf{T}&=\mathbb{1}-\frac{\mathbf{k}\cdot\mathbf{k}}{k^2}\end{aligned} \tag{16}$$

with

$$\mathbb{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{(unit vector)} \tag{17}$$

$\hat{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ is known to possess a high-frequency sum-rule expansion of the form

$$\hat{\sigma}^{\mathrm{H}1\mu\nu}(\mathbf{k}\omega) = -\sum_{\substack{l=1 \\ l=\text{odd}}} \frac{\hat{\bar{\Omega}}^{\mu\nu}_{l+1}(\mathbf{k})}{l+1} \tag{18}$$

$$\hat{\alpha}^{\mathrm{H}11\mu\nu}(\mathbf{k}\omega) = -\sum_{\substack{l=2 \\ l=\text{even}}} \frac{\hat{\bar{\Omega}}^{\mu\nu}_{l+1}(\mathbf{k})}{\omega^{l+1}} \tag{19}$$

as in the nonrelativistic case; the superscript H denotes "Hermitian part of"; prime and double prime denote "real part of" and "imaginary part of," respectively. As in the nonrelativistic case the $\hat{\bar{\Omega}}^{\mu\nu}(\mathbf{k})$ coefficients are obtained from the relation

$$\begin{aligned} \hat{\bar{\Omega}}^{\mu\nu}_{l+1}(\mathbf{k}) = (4\pi e^2/\hbar^{l-1}) \sum_{np} \{ & [\omega_{n0}(p, p-\hbar\mathbf{k}/2)]^{l-2} \\ & \times \langle 0 | \Pi^{\mu}_{\mathbf{k}}(\tau) | n \rangle \langle n | \Pi^{\nu}_{-\mathbf{k}}(0) | 0 \rangle - [-\omega_{n0}(p, p+\hbar\mathbf{k}/2)]^{l-2} \\ & \times \langle 0 | \Pi^{\nu}_{-\mathbf{k}}(0) | n \rangle \langle n | \Pi^{\mu}_{\mathbf{k}}(\tau) | 0 \rangle \}_{\tau=0} \end{aligned} \tag{20}$$

The high-frequency expansion of $\bar{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ is similar to that of $\hat{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ as given by equations (21) and (22) with $\bar{\Omega}^{\mu\nu}_{l+1}(\mathbf{k})$ replacing the corresponding $\bar{\Omega}^{\mu\nu}_{l+1}(\mathbf{k})$. The relationship between the two sets of coefficients up to $l=4$ is the same as for the nonrelativistic case.

The Hamiltonian of the system that satisfies equation (20) is given by

$$\begin{aligned} \mathbf{H} &= \sum_i \frac{m}{2} \mathbf{V}_1^2 + \frac{1}{2} \sum_{\substack{i \mp j \\ ij}} \mathbf{U}(|\mathbf{x}_i - \mathbf{x}_j|) \\ &= \sum_i \gamma^{-2} \frac{\pi_i^2}{2m} + \frac{1}{2} \sum_{\substack{ij \\ i \mp j}} \mathbf{U}(|\mathbf{x}_i - \mathbf{x}_j|) \end{aligned} \tag{21}$$

where $\mathbf{U}(|\mathbf{x}_i - \mathbf{x}_j|)$ is the velocity-independent interaction potential between a pair of particles.

We now turn to the calculation of the frequency moments (up to $l=4$). It is known that in an anisotropic system, in the presence of an external magnetic field, the dielectric tensor has six independent elements and, therefore $\bar{\alpha}^{\mu\nu}$ is nondiagonal. In this case both even and odd moments of $\bar{\Omega}^{\mu\nu}_{l+1}(\mathbf{k})$ exist. The real diagonal and off-diagonal elements satisfy the symmetrics condition

$$\bar{\Omega}^{\mu\nu}_{l+1}(\mathbf{k}) = \bar{\Omega}^{\nu\mu}_{l+1}(\mathbf{k}) \tag{22}$$

and the imaginary off-diagonal elements satisfy the antisymmetric condition

$$\bar{\Omega}^{\mu\nu}_{l+1}(\mathbf{k}) = -\bar{\Omega}^{\nu\mu}_{l+1}(\mathbf{k}) \tag{23}$$

as in the nonrelativistic case.

The first moment yields

$$\begin{aligned}\hat{\bar{\Omega}}^{\mu\nu}_{2}(\mathbf{k}) &= 4\pi e^2 \sum_{np} \left[\frac{\langle 0|\, \Pi^{\mu}_{\mathbf{k}}(\tau)\, |n\rangle\langle n|\, \Pi^{\nu}_{-\mathbf{k}}(0)\, |0\rangle}{\omega_{n0}(p,\, p+\hbar\mathbf{k}/2)} \right. \\ &\quad \left. + \frac{\langle 0|\, \Pi^{\nu}_{-\mathbf{k}}(0)\, |n\rangle\langle n|\, \Pi^{\mu}_{\mathbf{k}}(\tau)\, |0\rangle}{\omega_{n0}(p,\, p-\hbar\mathbf{k}/2)} \right]_{\tau=0} \\ &= \gamma^{-1}\omega_p^2 \mathbf{L}^{\mu\nu}_{k}\end{aligned} \tag{24}$$

where

$$\mathbf{L}^{\mu}_{\mathbf{k}} = \frac{k^{\mu}k^{\nu}}{k^2} \tag{25}$$

The second moment leads to

$$\begin{aligned}\hat{\bar{\Omega}}^{\mu\nu}_{3}(\mathbf{k}) &= \frac{4\pi e^2}{\hbar} \sum_{np} [\langle 0|\, \Pi^{\mu}_{\mathbf{k}}(\tau)\, |n\rangle\langle n|\, \Pi^{\nu}_{-\mathbf{k}}(0)\, |0\rangle \\ &\quad - \langle 0|\, \Pi^{\nu}_{-\mathbf{k}}(0)\, |n\rangle\langle n|\, \Pi^{\mu}_{\mathbf{k}}(\tau)\, |0\rangle]_{\tau=0} \\ &= \frac{2\pi e^2}{\hbar} [\langle 0|\, [\Pi^{\mu}_{\mathbf{k}}(\tau), \Pi^{\nu}_{-\mathbf{k}}(0)]\, |0\rangle - |[\Pi^{\nu}_{-\mathbf{k}}(0), \Pi^{\mu}_{\mathbf{k}}(\tau)]\, |0\rangle]_{\tau=0} \\ &= i\gamma^{-2}\omega_p^2 \frac{eB^{0\alpha}_{\eta}}{mc}\, \varepsilon^{\mu\nu\alpha}\end{aligned} \tag{26}$$

The third moment is given by

$$\begin{aligned}\hat{\bar{\Omega}}^{\mu\nu}_{4}(\mathbf{k}) &= \frac{4\pi e^2}{\hbar^2} \sum_{np} \left[\omega_{n0}\left(p,\, p-\frac{\hbar\mathbf{k}}{2}\right) \langle 0|\, \Pi^{\mu}_{\mathbf{k}}(\tau)\, |n\rangle\langle n|\, \Pi^{\nu}_{-\mathbf{k}}(0)\, |0\rangle \right. \\ &\quad \left. + \omega_{n0}\left(p,\, p+\frac{\hbar\mathbf{k}}{2}\right) \langle 0|\, \Pi^{\nu}_{-\mathbf{k}}(0)\, |n\rangle\langle n|\, \Pi^{\mu}_{\mathbf{k}}(\tau)\, |0\rangle \right]_{\tau=0}\end{aligned}$$

$$= \frac{2\pi e^2}{\hbar^2} \langle 0| \, [[\Pi^{\mu}_{\mathbf{k}}(\tau), H], \Pi^{\nu}_{-\mathbf{k}}(0)] + [[\Pi^{\nu}_{-\mathbf{k}}(0), H], \Pi^{\mu}_{\mathbf{k}}(\tau)] \, |0\rangle \, |_{\tau=0}$$

$$= \gamma^{-4} \frac{\omega_p^2 e B_\eta^0}{2mc} k^\alpha \langle 0| \, \varepsilon^{\mu\eta\nu} \frac{\partial}{\partial x^\alpha} + \varepsilon^{\mu\eta\alpha} \frac{\partial}{\partial x^\nu} + \varepsilon^{\alpha\eta\nu} \frac{\partial}{\partial x^\mu}$$

$$+ i\varepsilon^{\mu\eta\nu} \frac{eB_\eta^0}{2mc} (x^\nu - x^\mu) \, |0\rangle - \gamma^{-4} \frac{\omega_p^2 e B_\eta^0}{2mc} k^\mu \varepsilon^{\alpha\eta\nu}$$

$$\times \, \langle 0| \, \frac{\partial}{\partial x^\alpha} + i \frac{eB_\eta^0}{2mc} \chi^\nu \, |0\rangle$$

$$- \gamma^{-4} \frac{\omega_p^2 e B_\eta^0}{4mc} k^\nu \varepsilon^{\mu\eta\alpha} \langle 0| \, \frac{\partial}{\partial x^\alpha} - i \frac{eB_\eta^0}{2mc} \chi^\mu \, |0\rangle$$

$$- \gamma^{-4} \frac{\omega_p^2 e B_\eta^0}{2mc} k^\alpha k^\mu \langle 0| \, 2mc(eB_\eta^0)^{-1} \frac{\partial^2}{\partial \chi^\alpha \, \partial \chi^\nu} + i\varepsilon^{\alpha\eta\beta} \chi^\beta \frac{\partial}{\partial \chi^\nu}$$

$$- i\varepsilon^{\alpha\eta\nu} \chi^\alpha \frac{\partial}{\partial \chi^\alpha} - \varepsilon^{\mu\eta\alpha} \frac{eB_\eta^0}{2mc} (x^\mu)^2 \, |0\rangle$$

$$- \gamma^{-4} \omega_p^2 \frac{eB_\eta}{2mc} k^\alpha k^\nu \langle 0| \, 2mc(eB_\eta^0)^{-1} \frac{\partial^2}{\partial \chi^\alpha \, \partial \chi^\mu} + i\varepsilon^{\alpha\eta\beta} \chi^\beta \frac{\partial}{\partial \chi^\mu}$$

$$- i\varepsilon^{\alpha\eta\mu} \chi^\alpha \frac{\partial}{\partial \chi^\alpha} - \varepsilon^{\mu\eta\alpha} \frac{eB_\eta^0}{2mc} (x^\mu)^2 \, |0\rangle$$

$$- \gamma^{-4} \omega_p^2 \frac{eB_\eta}{2mc} k^\alpha k^\alpha \langle 0| \, 2mc(eB_\eta^0)^{-1} \frac{\partial^2}{\partial \chi^\mu \, \partial \chi^\nu} - i\varepsilon^{\alpha\eta\nu} \chi^\alpha \frac{\partial}{\partial \chi^\mu}$$

$$- i\varepsilon^{\alpha\eta\mu} \chi^\alpha \frac{\partial}{\partial \chi^\nu} + [\varepsilon^{\mu\eta\alpha} (\chi^\alpha)^2 \, \delta^{\mu\nu} - \varepsilon^{\mu\eta\nu} \chi^\mu \chi^\nu] \frac{eB_\eta^0}{2mc} \, |0\rangle$$

$$+ \gamma^{-2} \omega_p^4 \langle 0| \, \mathbf{L}_k^{\mu\nu} + \frac{1}{N} \sum_q \mathbf{L}_q^{\mu\nu} (S_{\mathbf{k}-\mathbf{q}} - S_{\mathbf{k}}) \, |0\rangle \tag{27}$$

The fourth moment leads to

$$\hat{\Omega}_5^{\mu\nu}(\mathbf{k}) = \frac{4\pi e^2}{\hbar^3} \sum_{np} \left\{ \left[\omega_{n0} \left(p, p - \frac{\hbar k}{2} \right) \right]^2 \langle 0| \, \Pi^{\mu}_{\mathbf{k}}(\tau) \, |n\rangle \langle n| \, \Pi^{\nu}_{-\mathbf{k}}(0) \, |0\rangle \right.$$

$$\left. - \left[-\omega_{n0} \left(p, p + \frac{\hbar k}{2} \right) \right]^2 \langle 0| \, \Pi^{\nu}_{-\mathbf{k}}(0) \, |n\rangle \langle n| \, \Pi^{\mu}_{\mathbf{k}}(\tau) \, |0\rangle \right\}_{\tau=0}$$

$$= \frac{4\pi e^2}{\hbar^3} \langle 0| \, [[[\Pi^{\mu}_{k}(\tau), \mathbf{H}], \mathbf{H}], \Pi^{\nu}_{-k}(0)]$$

$$- [[[\Pi^{\nu}_{-k}(0), \mathbf{H}], \mathbf{H}], \Pi^{\mu}_{-k}(0)] \, |0\rangle \, |_{\tau=0}$$

$$= -\gamma^{-6} \frac{\omega_p^2 eB_\eta^0}{4mc} k^\alpha \langle 0| \, \varepsilon^{\mu\eta\nu} \delta^{\mu\nu} \chi^\mu \frac{(eB_\eta^0)^2}{4m^2c^2}$$

$$+ \frac{7}{4} \varepsilon^{\mu\eta\nu} \frac{eB_\eta^0}{mc} \chi^\mu \frac{\partial}{\partial \chi^\nu} + \varepsilon^{\nu\eta\alpha} \frac{eB_\eta^0}{8m^2c^2} (x^\nu)^2 \frac{\partial}{\partial \chi^\mu}$$

$$+ \varepsilon^{\mu\eta\alpha} \frac{(eB_\eta^0)^2}{8m\hbar c} (\chi^\mu)^2 \frac{\partial}{\partial \chi^\nu} + i\varepsilon^{\mu\nu\eta} \frac{(eB_\eta^0)^3}{16m^2\hbar c^2} (\chi^\mu)^3$$

$$+ i6\varepsilon^{\mu\alpha\eta} \delta^{\mu\nu} \frac{eB_\eta^0}{mc} \frac{\partial}{\partial \chi^\alpha} |0\rangle - \gamma^{-6} \frac{\omega_p^2 eB_\eta^0}{4mc} k^\alpha k^\mu \langle 0| \, i \frac{7}{4} \varepsilon^{\mu\alpha\nu} \frac{\partial^2}{\partial \chi^\alpha \, \partial \chi^\alpha}$$

$$+ \frac{17}{8} \varepsilon^{\nu\eta\alpha} \frac{eB_\eta^0}{mc} \chi^\nu \frac{\partial}{\partial \chi^\alpha} + i \frac{7}{4} \varepsilon^{\nu\eta\alpha} \frac{eB_\eta^0}{m^2c^2} (x^\nu)^2 \, |0\rangle$$

$$- \gamma^{-6} \frac{\omega_p^2 eB_\eta^0}{4mc} k^\alpha k^\nu \langle 0| \, i \frac{7}{4} \varepsilon^{\mu\alpha\eta} \frac{\partial^2}{\partial \chi^\alpha \, \partial \chi^\alpha} + \frac{17}{8} \varepsilon^{\mu\alpha\eta} \frac{eB_\eta^0}{mc} \chi^\mu \frac{\partial}{\partial \chi^\alpha}$$

$$+ i \frac{7}{4} \varepsilon^{\mu\alpha\eta} \frac{eB_\eta^0}{m^2c^2} (x^\mu)^2 \, |0\rangle - \gamma^{-6} \frac{\omega_p^2 eB_\eta^0}{4mc} k^\alpha k^\alpha \langle 0| \, i6\varepsilon^{\mu\eta\alpha} \frac{\partial^2}{\partial \chi^\alpha \, \partial \chi^\alpha}$$

$$+ i6\varepsilon^{\nu\alpha\eta} \frac{\partial^2}{\partial \chi^\alpha \, \partial \chi^\mu} + i \frac{3}{2} \varepsilon^{\mu\eta\nu} \frac{\partial^2}{\partial \chi^\alpha \, \partial \chi^\alpha} + \frac{3}{2} \varepsilon^{\mu\eta\alpha} \delta^{\mu\nu} \frac{eB_\eta^0}{mc}$$

$$+ 3\varepsilon^{\nu\eta\alpha} \frac{eB_\eta^0}{mc} \chi^\nu \frac{\partial}{\partial \chi^\nu} + 3\varepsilon^{\nu\eta\alpha} \frac{eB_\eta^0}{mc} \chi^\nu \frac{\partial}{\partial \chi^\mu} - i \frac{15}{4} \varepsilon^{\mu\eta\nu} \frac{eB_\eta^0}{m^2c^2} (\chi^\mu)^2$$

$$+ i\varepsilon^{\mu\nu} (\chi^\nu - \chi^\mu) \frac{eB_\eta^0}{mc} |0\rangle + i\gamma^{-4} \omega_p^4 \frac{eB_\eta^0}{2mc} \langle 0| \, \mathbf{L}_k^{\mu\nu}$$

$$+ \frac{1}{N} \sum_q (\varepsilon^{\mu\eta\alpha} \mathbf{L}_q^{\alpha\nu} + \varepsilon^{\alpha\eta\nu} \mathbf{L}_q^{\alpha\mu})(S_{\mathbf{k}-\mathbf{q}} - S_{\mathbf{k}}) \, |0\rangle \tag{28}$$

In order to obtain an explicit expression for $\hat{\hat{\Omega}}^{\mu\nu}_{l+1}(\mathbf{k})$, I choose the k system in which (Genga, 1988b)

$$\begin{aligned} \mathbf{k} &= (0, 0, k) \\ B_x^0 &= B^0 \sin\theta \\ B_y &= 0 \\ B_z &= B^0 \cos\theta \end{aligned} \tag{29}$$

and

$$\begin{aligned} q_x &= q \sin\theta \cos\theta \\ q_y &= q \sin\theta \sin\theta \\ q_z &= q \cos\theta \end{aligned} \tag{30}$$

The components of the external magnetic field given by equation (29) are obtained when the Landau gauge $A^0 = \frac{1}{2}(0, B_z^0 x - ZB_x^0, 0)$ is applied.

3. LONG-WAVELENGTH LIMIT

In the long-wavelength limit, equations (27)–(31) lead to the following elements of the frequency moments:

$$
\begin{aligned}
\hat{\hat{\Omega}}_2^{11} &= \hat{\hat{\Omega}}_2^{22} = 0 \\
\hat{\hat{\Omega}}_2^{33} &= \gamma^{-1}\omega_p^2 \\
\hat{\hat{\Omega}}_3^{12}(\mathbf{k}) &= -\hat{\hat{\Omega}}_3^{21}(\mathbf{k}) = i\gamma^{-2}\omega_p^2\Omega\cos\theta \\
\hat{\hat{\Omega}}_3^{23}(\mathbf{k}) &= -\hat{\hat{\Omega}}_2^{32}(\mathbf{k}) = i\gamma^{-2}\omega_p^2\Omega\sin\theta \\
\hat{\hat{\Omega}}_4^{11}(\mathbf{k}) &= -\frac{2}{15}\frac{\omega_p^2}{m}E_{\text{corr}}k^2 \\
\hat{\hat{\Omega}}_4^{13}(\mathbf{k}) &= 0 \\
\hat{\hat{\Omega}}_4^{22}(\mathbf{k}) &= -\frac{2}{15}\gamma^{-2}\frac{\omega_p^2}{m}E_{\text{corr}}k^2 \\
\hat{\hat{\Omega}}_4^{33}(\mathbf{k}) &= \gamma^{-2}\omega_p^4 - \gamma^{-4}\frac{\omega_p^2}{m}\left(\frac{3P_{\text{F}}^{(0)^2}}{m} - \frac{4}{15}\gamma^2 E_{\text{corr}}\right)k^2 \\
\hat{\hat{\Omega}}_5^{12}(\mathbf{k}) &= -\hat{\hat{\Omega}}_5^{21}(\mathbf{k}) = -i\gamma^{-6}\frac{\omega_p^2\Omega}{8m}\left(\frac{3P_{\text{E}}^{(0)^2}}{m} + \frac{16}{15}\gamma^2 E_{\text{corr}}\right)k^2\cos\theta \\
\hat{\hat{\Omega}}_5^{23}(\mathbf{k}) &= -\hat{\hat{\Omega}}_5^{21}(\mathbf{k}) = -i\gamma^{-6}\frac{\omega_p^2\Omega}{8m}\left(\frac{15P_{\text{F}}^{(0)^2}}{m} - \frac{24}{15}\gamma^2 E_{\text{corr}}\right)k^2\sin\theta
\end{aligned}
\tag{31}
$$

where $|0\rangle$ is of the form (Genga, 1988b)

$$|0\rangle = (2\pi)^{-1/2}\,\lambda^{-1}e(y-y_0)^2/4\lambda^2 + iP_2/\hbar \tag{32}$$

with

$$
\begin{aligned}
\lambda &= \frac{\hbar}{m\Omega} \\
y_0 &= -\frac{2cp_x}{e} \\
y &= B_z x - B_x z = -\frac{cp_y}{e} \\
\Omega &= -\frac{eB^0}{mc}
\end{aligned}
\tag{33}
$$

4. RELATIVISTIC EFFECT

The relativistic effect on the undamped high-frequency, quasi-one-dimensional quantum plasma waves at $T=0$ K is determined in this section by using the high-frequency sum rules (HFSRs). The high-frequency modes of interest are the "ordinary" and "extraordinary" modes propagating both along and across the external magnetic field; the extraordinary mode under consideration is the one with cutoff frequency $\omega_2 = \frac{1}{2}\Omega[1+(1+4\omega_p^2/\Omega^2)^{1/2}]$.

4.1. Propagation Parallel to Magnetic Field

It is known (Genga, 1988b, 1989) that here only the longitudinal and the extraordinary modes exist.

4.1.1. Longitudinal Mode

The behavior of longitudinal plasmons is known to be determined by the dispersion relation

$$\varepsilon_{33}(k\omega) = 1 + \alpha_{33}(k\omega) = 0 \tag{34}$$

When a small perturbation is applied to the dispersion relation the ensuing plasmon frequency is obtained to be of the form

$$\omega^2(\mathbf{k}) = \gamma^{-1}\omega_p^2\left[1 - \gamma^{-5/2}\frac{\omega_p^2}{m}\left(6E_{\mathrm{F}} - \frac{4}{15}\gamma^2 E_{\mathrm{corr}}\right)k^2\right] \tag{35}$$

where E_{F} is the lowest Landau level nonrelativistic particle kinetic energy defined as

$$E_{\mathrm{F}} = \frac{P_{\mathrm{F}}^{(0)2}}{2m} \tag{36}$$

The correlation term is seen to be of the order γ^2 greater than the quantum term.

4.1.2. Extraordinary Mode

The dispersion relation that determines the behavior of the extraordinary mode is known to be of the form

$$[\varepsilon_{11}(\mathbf{k}\omega) - n^2]^2 - \varepsilon_{12}^2(\mathbf{k}\omega) = 0 \tag{37}$$

Equation (37) leads to the ensuing frequency of the form

$$\omega^2(\mathbf{k}) = \gamma^{-1}\omega_2^2\left[1 + \frac{2}{3}\gamma^{1/2}\left(\frac{c_2^2}{\omega_2^2} + 2\,\frac{\gamma^{-2}\omega_p^2}{15m\omega_2^4}\,E_{\text{corr}}\right)k^2\right] \tag{38}$$

It can still be seen that the correlations enhance the positive refractive dispersions for finite **k**; however, the correlation term is now smaller than the refractive term by a relativistic factor of γ^{-2}. Further, it can be seen that the ensuing frequency of oscillations is reduced by the relativistic factor as given by equation (35) and (38), respectively.

4.2. Propagation Perpendicular to Magnetic Field

In this case it is known that only a pure transverse mode, called the "ordinary mode," and a coupled transverse–longitudinal mode, known as the "extraordinary mode," exist. The dispersion relation for the ordinary mode is given as

$$\varepsilon_{11}(\mathbf{k}\omega) - n^2 = 0 \tag{39}$$

whereas that for the extraordinary mode is given by

$$[\varepsilon_{22}(\mathbf{k}\omega) - n^2]\,\varepsilon_{33}(\mathbf{k}\omega) - \varepsilon_{23}^2(\mathbf{k}\omega) = 0 \tag{40}$$

4.2.1. Ordinary Mode

When a small perturbation is applied to the dispersion relation it is found that there is no shift in frequency due to correlations.

4.2.2. Extraordinary Mode

In this case the ensuing frequency of oscillation after a small perturbation is applied to the dispersion relation is given by

$$\omega^2 = \gamma^{-1}\omega_2^2\left\{1 + \gamma^{1/2}\left[\frac{c^2}{\omega_p^2} - \frac{2\gamma^{-4}}{m\omega_2^2}\left(3E_{\text{F}} - \frac{1}{15}\gamma^2 E_{\text{corr}}\right)\right]k^2\right\} \tag{41}$$

The correlation term is seen to enhance the negative quantum dispersion for finite k as in the nonrelativistic case. However, the correlation is now larger than the quantum term by a relativistic factor of γ^2. Further, the ensuing frequency of oscillation is reduced by the relativistic factor as shown in equation (41).

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